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Edwards measure and the steady-state regime of a model with kinetic constraints under tapping

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Abstract

We study the tapping dynamics of a one-dimensional Ising model with symmetric kinetic constraints. We define and test a variant of the Edwards hypothesis that one may build a thermodynamics for the steady state by using a flat measure over the metastable states with several macroscopic quantities fixed. Various types of tapping are compared and the accuracy of this measure becomes quickly excellent when the number of quantities fixed on average increases, independent of the way the system is excited. We attribute the validity of the naive flat measure at weak tapping to the spatial separation of density defects.

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1. Introduction

The physical properties of granular materials have been extensively studied during the last few decades because of their wide presence in industry and their interest for fundamental statistical mechanics. In particular, since the compaction experiments of the Chicago group [1], there have been many attempts to understand the mechanism of compaction of dense powders under weak tapping and their steady-state behaviour. In these systems, energy is completely dissipated after each tap and the thermal fluctuations are negligible compared to the gravitational energies involved. Such a system evolves from one blocked state to another due to the external driving without obvious detailed balance and the usual tools of statistical mechanics have to be generalized. Edwards and co-workers made the assumption that in this context it is possible to build a 'thermodynamics' by using a flat measure over the blocked states in the steady state, the main ingredient being that all blocked, or metastable, configurations are equiprobable in the steady state [2]. This is the simplest and most natural first assumption. This ergodicity in the steady state could conceivably arise from the extensive, non-local nature of the tapping dynamics. It seems to be a rather hard task to define and

compute the entropy of blocked states in a realistic granular medium, for example an assembly of grains in a box [3]. Hence the Edwards measure has been recently put to test on a wide variety of simple models. It was found in the three-dimensional Kob–Andersen [4] and Tetris models [4, 5] that the flatness assumption is good for dense systems, and also in the context of partially analytically tractable one-dimensional models [6, 7] and spin models on random graphs [8, 9]. Recent simulations on three-dimensional sheared packing of spheres [10] have applied the Edwards measure to a realistic model and opened up the possibility of testing Edwards' hypothesis on physical systems. Moreover, a tapping mechanism has been introduced in spin glass models and the Edwards measure was shown to be very efficient in describing phase transitions in the steady state [11]. As the Edwards measure seemed to give very good results on the thermodynamics of the tapped Ising chain [6], it has also been tested on one-dimensional kinetically constrained models [12]. Variants of these models were studied to test the Stillinger and Weber idea [13, 14], which is to decompose the space of configurations into valleys, to project each valley onto its minimum (called inherent structure) and to reduce the dynamics of the system to a dynamics on inherent structures. Ising models with kinetic constraints allow one to test this decomposition in the following way [14]: two models differ only in the constraints of the dynamics and share the same local energy minima. As the low temperature the dynamics are different, the dynamics cannot be reduced to a simple sampling of inherent structures. For the same reason, the Edwards measure was expected to fail. Berg *et al* [12] submitted these models to two kinds of tapping, which they called 'thermal' and 'random' and found that the Edwards measure fails, as demonstrated by De Smedt *et al* in the limit of large tapping [15]. Moreover, they argued that the validity of the flatness assumption depends on the tapping mechanism, that is to say the way energy is injected into the system. In this paper, we show how the basic Edwards measure can be generalized to build the thermodynamics of the asymptotic regime and that the validity of this measure is independent of the tapping mechanism. We shall attribute the deviation of the measure sampled during tapping simulations from the Edwards one to short distance correlations in the metastable states and try to explain why the basic flat measure is efficient at very weak tapping.

2. The generalized Edwards measure

Edwards' hypothesis consists in assuming that the steady-state dynamics is ergodic, i.e. the resulting measure over blocked configurations is flat. In addition, if some quantities are conserved on average, the measure must be restricted to the blocked configurations having these average quantities, as is the case in ordinary statistical physics. Other quantities, which are not conserved, fluctuate around a value which maximizes the Edwards entropy. The original idea of Edwards and co-workers [2] was that an assembly of grains in a gently vibrated box is fully characterized by its density (or volume 'V'), which is the only quantity to fix on average in the steady state. Hence they introduced, as a Lagrange multiplier, a 'compactivity':

$$X_{\mathrm{Edw}}^{-1} = \frac{\partial S_{\mathrm{Edw}}}{\partial V}. \tag{1}$$

However, there is no evidence that only one quantity such as the density (or the energy for spin systems) has a nonzero Lagrange multiplier and, as already mentioned [12], at least two quantities should be fixed on average in order to describe the steady state with a flat measure.

Let us then build a generalized Edwards measure and imagine a granular-like system submitted to external forcing whose effect is to move the system from one blocked

configuration to another. Let us assume that the balance between forcing and relaxation is such that in the asymptotic stationary regime exactly m quantities X_k ($k = 1, \ldots, m$) are conserved on average. For instance, for a mixture of hard spheres of different diameters d_1 and d_2, one can consider $X_1 = h_1$ and $X_2 = h_2$ the mean heights of each kind of sphere [16]. We introduce corresponding Lagrange multipliers β_k and compute the grand canonical partition function:

$$Z(\{\beta_k\}) = \int \prod_k \mathrm{d}X_k \, \mathrm{e}^{-\sum_k \beta_k X_k + S(\{X_k\})} \tag{2}$$

where $S(\{X_k\})$ is the entropy of the blocked configurations restricted to that of given $\{X_k\}$. In the limit of a large volume or number of particles, the integrand is sharply peaked around one value $\{X_k^*\}$ which maximizes $-\sum_k \beta_k X_k + S(\{X_k\})$. The Lagrange multipliers are given as in usual statistical mechanics by

$$\beta_k = \frac{\partial S}{\partial X_k} \tag{3}$$

and the average of X_k is

$$\langle X_k \rangle = -\frac{\partial \log Z}{\partial \beta_k} = X_k^*. \tag{4}$$

2.1. The model

The model we shall consider in this paper is a variant of the Fredrickson–Anderson (FA) model, which will be referred to in what follows as the symmetrically constrained Ising model (SCIM). In the original FA model [17], particles are disposed on a one-dimensional lattice. At each site i is associated with its occupation number $n_i = 0, 1$. The total energy is $-\sum_i n_i$ and the dynamics is constrained, that is the usual metropolis probability for a spin to flip is weighted by an acceptance ratio: $W(n_i \to 1 - n_i) = \frac{1}{2}(2 - n_{i-1} - n_{i+1}) \min(1, \mathrm{e}^{-\beta \Delta E})$. In this model, equilibration proceeds through elimination of isolated holes by coalescence, which is slower and slower at low temperature, as these defects are very separated, and the dynamics and the system undergo a dynamical glass transition [17]. This model has been studied in the context of granular compaction [7]. It was shown to have very slow dynamics consistent with the inverse logarithmic law found in experiments [1], followed by a steady state well described by a flat measure over the blocked states. Here, the kinetic constraint will be changed a little: in any single move, a particle can be added to or removed from a site only if at least one of the neighbouring sites is empty. It has been recently shown that a basic application of the Edwards measure is unable to describe the thermodynamics of the steady state of the SCIM subjected to two types of tapping [12]:

(i) *'random':* occupation of each site is changed with probability $p \in [0, 1/2]$;
(ii) *'thermal':* one Monte Carlo sweep is made, with Metropolis probability $p(n_i \to 1 - n_i) = (1 - n_{i-1} n_{i+1}) \min(1, \mathrm{e}^{-\beta \Delta E})$. This thermal tapping was introduced and studied in spin models [9] and lattice models [18] of granular matter, after the analogy between vibration and thermal noise was pointed out in [19].

In between taps, the system undergoes a zero temperature dynamics which corresponds to adding particles at empty sites having at least one empty neighbour, until it becomes blocked in a metastable state. This dynamics can, however, be seen as the zero temperature Glauber dynamics of a model (without kinetic constraints) with energy per site $E = \frac{1}{N} \sum_i ((1 - n_i)(1 - n_{i-1} n_{i+1}) - n_{i-1} n_{i+1})$, where only moves which strictly lower the energy

are allowed. With this definition, the metastable states are now energetically metastable. The contributions of site i to this energy are

- -1, if site $i-1$ and $i+1$ are occupied;
- 0, if site i is occupied and either site $i-1$ or site $i+1$ is empty;
- 1, if site i is empty and either site $i-1$ or site $i+1$ is empty.

In addition, in the following, the average occupation $\rho = \frac{1}{N}\sum_i n_i$, which involves no interactions, will be called 'density'. This point is fundamental, as we have to keep in mind that the blocked states are reached by 'gradient' descent in the energy landscape. Hence, as the basins of attraction of the metastable states are not *a priori* the same, the assumption that the latter are sampled in a flat manner is a strong one.

As in granular media the complexity emerges from the kinetic constraints due to hard-core repulsion and collisions, this model is thus a simple one-dimensional granular medium subjected to tapping, with an energy E driving the 'falling' of the particles and a density ρ characterizing the compactness. The definition of the entropy of metastable states is exactly that of Edwards and one can apply Edwards' hypothesis in its original spirit.

The zero temperature dynamics stops when all empty sites are isolated. This gives a simple characterization of any metastable state as a sequence of domains of neighbouring occupied sites, separated by one empty site, and allows one to predict easily the entropy or the distribution $P(l)$ of domain sizes among the metastable states. Our goal is then to compute as many characteristics of the steady state as possible, with a minimal set of quantities obtained by measurement, and if possible to find some circumstances where the simplest Edwards measure is a good approximation.

The description of the steady-state regime by the Edwards measure fails in the regime of low density, where the average length of the domains is small. On the other hand, it seems to be fairly accurate in the high density regime, where large domains dominate. Moreover, the zero temperature dynamics involves short range interactions, so we expect that, in the blocked states sampled by the tapping dynamics, correlations at long distances are induced by correlations at short distances. In the context of the SCIM, Edwards' hypothesis implies that all correlations are obtained from the average domain length. If one wants to improve the measure by introducing new Lagrange multipliers, one can add some which fix the average value of short length scale characteristics, such as the number of domains of length one or two.

For simplicity, and as the density ρ and the energy per site E are natural quantities of the model, we have computed the entropy and the distribution of domain lengths in the Edwards ensemble for given values of ρ, E and the probability $P(l < 3)$ that a domain has length smaller than 3, which are linear combinations of ρ, $P(l = 1)$ and $P(l = 2)$, so short length scales are fixed on average, as explained above. To keep only two quantities, we maximize the Edwards entropy with respect to $\alpha = P(l < 3)$ with ρ and E fixed. Maximizing again with respect to E gives the simplest Edwards measure. The results are given in the appendix.

3. Numerical simulations

Here, we shall compare the accuracy of different generalizations of the flat measure in numerical simulations of tapping, as well as the influence of the tapping mechanism. In order to clarify how different excitations can lead to different regimes of density, it is important to separate the ingredients of the tapping (ii), that is the kinetic constraints and the thermal condition. In addition, we can combine the kinetic constraints with the random tapping (i).

So we define four tapping mechanisms, depending on whether, under tapping, the kinetic constraints are respected and whether the tapping is random:

(RU) *'Random unconstrained'*: occupation of each site is changed with probability $p \in [0, 1]$. Note that there is no reversal symmetry as in ± 1 Ising spin systems, so p can be greater than $1/2$;

(RC) *'Random constrained'*: one Monte Carlo sweep is made during which the occupation of each chosen site is changed randomly with probability $p \in [0, 1]$ if it has an empty neighbouring site;

(TU) *'Thermal unconstrained'*: one Monte Carlo sweep is made with Metropolis probability $p(n_i \to 1 - n_i) = \min(1, \mathrm{e}^{N\Delta\rho/T_\rho})$, T_ρ being the tunable intensity of tapping and $\Delta\rho = \frac{1-2n_i}{N}$ the variation of the density during the Metropolis step;

(TC) *'Thermal constrained'*: one Monte Carlo sweep is made with Metropolis probability $p(n_i \to 1 - n_i) = (1 - n_{i-1}n_{i+1}) \min(1, \mathrm{e}^{N\Delta\rho/T_\rho})$;

In the two latter cases, we use ρ instead of E in order to compare with the results of Berg *et al* [12] (so (RU) corresponds to (i) and (TC) corresponds to (ii)). (However, this is equivalent in the (TC) case and the results are not qualitatively changed if we use E instead of ρ in (TU).) We have carried out simulations for each of the four tapping mechanisms above. The systems had $N = 10^5$ and $N = 10^6$ spins and several quantities have been recorded during 10^6 taps once the steady state is reached:

- the energy E and the density ρ;
- the distribution of the domain sizes $P(l) = \text{Probability}(\text{size} = l)$;
- the fluctuations of E and ρ:

$$c_E = N(\langle E^2 \rangle - \langle E \rangle^2) \tag{5}$$

$$c_\rho = N(\langle \rho^2 \rangle - \langle \rho \rangle^2). \tag{6}$$

As expected, E does not maximize the entropy when ρ only remains fixed, and a small but significant dependence of the curve of E versus ρ on the tapping mechanism is observed.

In order to put to test the applicability of the canonical ensemble with three nonzero Lagrange multipliers, we shall compare the distribution $P(l)$ of domain lengths and the fluctuations of ρ and E recorded during the simulations, with their corresponding values in the Edwards ensemble restricted to the configurations where energy, density and probability for domain length to be at most three, are equal to that measured during the simulations. In the following, we shall refer to the corresponding measure as M3. The same procedure is carried out for two and one nonzero Lagrange multipliers, where the ensemble was restricted to energy and density, or density only, with the corresponding measures referred to as M2 and M1, respectively.

Let us remark that if one assumes that the distributions of the lengths of two neighbouring domains are independent, $P(l)$ is enough to compute all correlation functions involving a finite number of sites, so measuring c_ρ and c_E may be redundant. However, these fluctuations involve a large number of terms, and so are very sensitive to the deviations to the exact measure sampled during the simulation. As we shall see below, the comparisons between the different generalized measures and the numerically generated one are much more convincing when comparing the fluctuations than when comparing the distributions of domain lengths.

In figures 1 and 2, the computation of the fluctuations of the density obtained from the tapping simulations are displayed and compared to the ones expected from measures M1, M2 and M3, as a function of the steady-state density. As explained in the next section, different kinds of tapping cover different energy and density ranges, so we can test the generalized

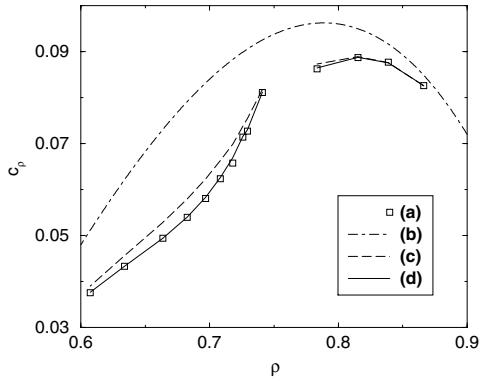

Figure 1. Fluctuations of the density versus density for the mechanisms (RU) and (RC): numerical computation in the steady state (*a*), computation by using a flat measure with one quantity fixed (*b*), two quantities fixed (*c*) and three quantities fixed (*d*). The left part of (*a*), (*c*) and (*d*) has been obtained by using the values of E, ρ and α recorded during (RU) and the right part during (RC).

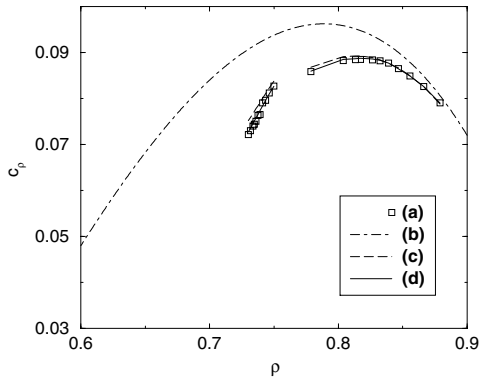

Figure 2. Fluctuations of the density versus density for the mechanisms (TU) and (TC): numerical computation in the steady state (*a*), computation by using a flat measure with one quantity fixed (*b*), two quantities fixed (*c*) and three quantities fixed (*d*). The left part of (*a*), (*c*) and (*d*) has been obtained by using the values of E, ρ and α recorded during (TU) and the right part during (TC).

Edwards measure on a wide range of energies or densities. As is already known [12], if only ρ is fixed, this measure is accurate only at high density. We have verified that the measure with only E fixed works only at low energy. Fixing both ρ and E gives quite good results, but there is still a difference between the tapping simulations and the value expected from the generalized Edwards measure. The distribution of domain lengths $P(l)$ obtained with (TC) with $T_\rho = 1.3$ is shown in figure 3 and with (RU) with $p = 0.4$ in figure 4. The non-exponential behaviour of $P(l)$ at short lengths indicates that we have to fix at least two quantities on average. The computation of $P(l)$ using the measure M2 is better than that using M1, but a difference with the simulations remains at low density, as shown in figure 3. However, $P(l)$ becomes exponential as soon as $l \geqslant 3$, which indicates that the large scale degrees of liberty maximize the entropy, so that only three parameters should be enough to describe the whole distribution $P(l)$. So, the computation using the measure M3 is expected to predict with accuracy the fluctuations of density and the distribution of domain lengths, as is the case in figures 1–3. In addition, we remark that the local minimization of energy involves three consecutive sites, so that the effective interaction length due to the kinetic constraints is

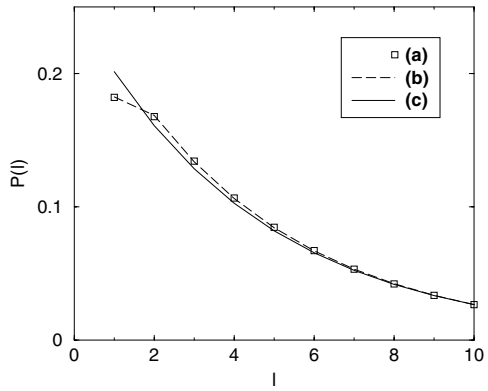

Figure 3. Distribution of the domain lengths obtained with (TC) for $T_\rho = 1.3$. The numerical computation in the steady state (*a*) is indistinguishable from the analytical calculation using the Edwards measure with two quantities fixed (*b*) but differs from the analytical calculation with only ρ fixed (*c*).

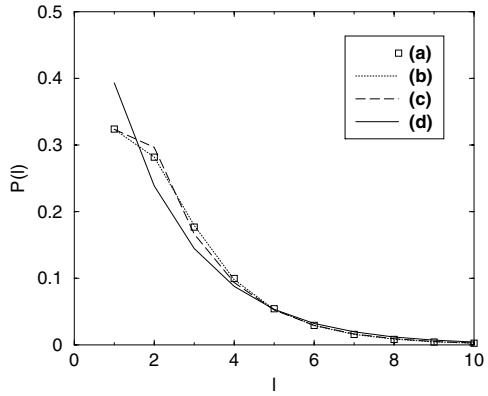

Figure 4. Distribution of the domain lengths obtained with (RU) for $p = 0.4$. The numerical computation in the steady state (*a*) is indistinguishable from the analytical calculation using the Edwards measure when three quantities are fixed (*b*) and starts to differ from it when two (*c*) or one (*d*) only are fixed.

three. So one can expect that if the zero temperature dynamics involves four consecutive sites, the measure M4 will be needed.

In such models, where energy is injected into the system by external forcing, there is no conservation law to ensure that any given quantity must be fixed on average as a result of the equilibration between the internal relaxation into metastable states and the external driving. However, as expected, the generalized Edwards measure converges to the measure sampled during the tapping when the number of Lagrange multipliers increases. Moreover, the convergence to the original Edwards measure is more rapid as the tapping intensity is lowered and near the maximum of density, the simplest measure gives a very good approximation. At low tapping, if we consider the tapping mechanisms (TC) and (RC), which allow one to reach this regime of high density (see section 4), the average domain size is large and the dynamics is dominated by the diffusion of small sequences of short domains, separated by long domains. In the language of granular media, the mobile particles are localized in regions

of weak density, which are far from one another at high density and then diffuse independently. This is reminiscent of similar results in the context of the Kob–Andersen model, for which Edwards' hypothesis was found to apply at high density. In these models, as well as in granular media, in the very compact regime, the majority of particles are unable to move during the taps because of the hard-core constraints.

Hence, we conjecture that this scenario is more general: let us consider a granular-like system, that is an assembly of hard 'heavy' particles, evolving among blocked configurations thanks to a macroscopic forcing. If the driving excitation is weak enough so that in the steady state the defects (regions where the density is low) are distant from each other, the dynamical measure over the blocked configurations is flat. In contrast, if the system is near the random loose packing, it is very heterogeneous in space and a majority of particles is allowed to move, contributing to large avalanches which break the ergodicity.

4. Comparing different tapping mechanisms

It has been argued through the tapping mechanisms (i) and (ii) that 'thermal' tapping is much more efficient in sampling the configurations in the flat manner than 'random' tapping. Moreover, it was added [12] that the former allows the system to reach high densities, whereas the latter was confined below $\rho^* = 3/4$. Here we shall clarify this issue by separating the influence of the 'thermal' or 'random' nature of the excitation and the presence or absence of a kinetic constraint during the tap by comparing the results of the simulations with the mechanisms (TC), (RC), (TU) and (RU).

Indeed, if the same kinetic constraint as that of the zero temperature dynamics is not imposed during the tap, domains can split or coalesce as the system is excited, whereas the number of domains changes during the relaxation only through nucleation. Thus large domains are unstable with respect to (TU) and (RU) and stable with respect to (TC) and (RC). This remark allows one to compute the maximal steady-state density accessible to (TU) and (RU), obtained in the limit of zero excitation intensity. To do so, we assume that the tapping is so weak that in a given sequence of sites, only one site is changed. Neglecting the correlations of the lengths of consecutive domains, we focus on three consecutive domains, where at most one change occurs in the central one (which size is $\langle l\rangle = \sum_l l P(l)$) or at its frontier during a tap. The average density is given by $\rho = \langle l\rangle/(\langle l\rangle + 1)$ and its variation after one tap (e.g. in the next metastable state) is

$$N\Delta\rho = ((N\rho + 1)p(a) + N\rho p(b) + (N\rho - 1)p(c)) - N\rho \tag{7}$$

where

$$p(a) = \frac{1}{\langle l\rangle + 1} \qquad p(b) = \frac{2}{\langle l\rangle + 1} \qquad p(c) = \frac{\langle l\rangle - 2}{\langle l\rangle + 1} \tag{8}$$

and $p(a)$, $p(b)$ and $p(c)$ are the probability of the contributions to the variation of the density displayed in figure 5. Gathering these three terms gives

$$N\Delta\rho = \frac{3 - \langle l\rangle}{1 + \langle l\rangle}. \tag{9}$$

Here, because any empty site is shared by two domains in a metastable state, we do not add a particle in the empty site at the left of the central domain to avoid redundancies and then the denominator in equation (9) is just the number of possible moves. This gives $\rho^* = 3/4$ in the large time limit, even if the system is prepared in a high density state.

In contrast, if the dynamics is constrained, the zero tapping limit corresponds to moving domain walls and then slowly eliminates small domains as in the zero temperature evolution

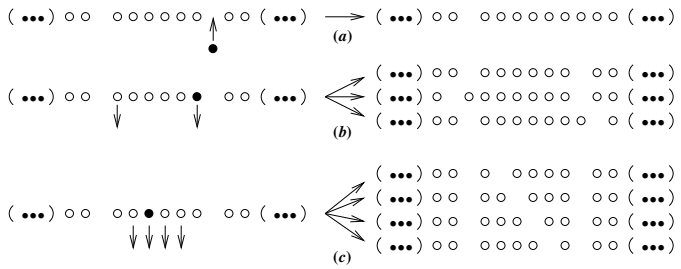

Figure 5. Variation of the density when one particle is added or removed at each tap. Here $\langle l \rangle = 6$. Black particles are those which are added or removed and the arrows indicate the other possible choices which give the same density in the final configuration. The left sequences represent a piece of the configuration before the tap and the right represent all the possible metastable configurations reached after the tap and the following zero temperature dynamics.

of the one-dimensional Ising model [20]. Hence, the mean length of the domain walls grows until it is of the order of the size of the system, and the density approaches very slowly the maximum value possible.

Now, it is clear from figures 1 and 2 that (TU) and (RU) on one hand, (TC) and (RC) on the other lead to comparable deviations of the measures M1 and M2 from the dynamical one. So, the validity of Edwards' hypothesis is *independent* of the way the system is tapped. This is not surprising, since all configurations are connected by the unconstrained part of the four tapping mechanisms (thermal or random) so that the ergodicity is broken by the kinetic constraint and the zero temperature dynamics. Hence, the only relevant difference between the four tapping mechanisms introduced here is whether or not the kinetic constraint is respected during the excitation. However, the only influence of this constraint on the accuracy of the Edwards measure is through the range of densities accessible.

Even if the measures M1, M2 or M3 are insensitive to the kind of excitation, we can use the results of the tapping simulations with mechanisms (TC) and (RC), or (TU) and (RU) to find whether the measure M1 is accurate or not, without knowing *a priori* the Edwards entropy. Indeed, as far as lattice models such as the SCIM involved here, the Edwards measure M1 can be computed at least numerically and compared to the one obtained dynamically. However, as far as realistic granular media are concerned, this is no longer possible. If we compare the values of the observables measured during the tapping simulations with (TC) and (RC), the fluctuations of density for instance, we find some small differences, in the regime of density where M1 does not apply. One can explain these differences by considering the measure M2. Indeed, if for instance the density and the energy are fixed, the value of the entropy in the steady state, when the tapping intensity decreases, is a path on a two-dimensional surface, which depends on the tapping mechanism: as the energy is injected in the system in two different manners, the Lagrange multipliers are not expected to be the same for the same value of the density. In contrast, as M1 involves only the density, characteristics of the steady state, such as density fluctuations, should not depend on the excitation mechanism for a given value of the steady-state density, if M1 applies. So one can try to imagine how the Edwards measure can be put to test experimentally. Let us assume that a given steady-state packing fraction ϕ of the same grains can be obtained by several forcing mechanisms, such as shear and vibration for instance. If the amplitude of the fluctuations of the packing fraction differs for different kinds of excitation, ϕ cannot be the only relevant macroscopic quantity.

5. Conclusion

In this paper, we have addressed the possibility of describing the steady-state regime of a simple model of granular matter by using a flat measure. In the high density regime, the knowledge of the mean length of domains was enough to give a qualitative description of global quantities, such as the fluctuations of ρ. However, Lagrange multipliers corresponding to the shortest length scales have been introduced in order to compute with accuracy all the quantities of interest for all the densities accessible. We have shown that by introducing a small number (related to the length scale of the effective interaction involved in the zero temperature dynamics) of multipliers, one may characterize the system in cases where the basic flat measure fails. Moreover, we have given a more general context where the latter applies. Its success in the weak tapping limit was attributed to the diffusion of the regions where the density is low, well separated in space. We then compared different tapping mechanisms and showed that the Edwards measure was indifferent to the way the system was excited. In addition, we have proposed a way of testing the original Edwards measure without any *a priori* information about the entropy of metastable states, by applying different kinds of tapping.

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Appendix

The entropy s and the distribution $P(l)$ of domain sizes in different generalized Edwards ensembles can be obtained by simple combinatorial arguments. Especially, if one imposes that a given domain has size l and counts the number of possible metastable states which fulfil this constraint (with ρ or E or α fixed), the leading term gives e^{Ns} and the term of order 1 gives $P(l)$.

A1. The original measure (M1)

The entropy of metastable states per site is

$$s(\rho) = -\rho \log \frac{2\rho - 1}{\rho} + (1 - \rho) \log \frac{2\rho - 1}{1 - \rho} \tag{A.1}$$

and the distribution of domain lengths is exponential:

$$P(l) = \frac{1 - \rho}{\rho} \left(\frac{2\rho - 1}{\rho} \right)^{l-1} \theta(l - 1) \tag{A.2}$$

where θ is the Heaviside step function.

A2. Fixing two quantities (M2)

The entropy of metastable states per site is

$$\begin{aligned} s(\rho, E) = {} & (1 - \rho) \log(1 - \rho) + (2\rho - 1) \log(2\rho - 1) - 2(\rho + E) \log(\rho + E) \\ & - (1 - 2\rho - E) \log(1 - 2\rho - E) - (\rho - 1 - E) \log(\rho - 1 - E) \end{aligned} \tag{A.3}$$

and the distribution of domain lengths is exponential only for $l \geqslant 2$:

$$P(l) = \left(\frac{1 - 2\rho - E}{\rho + E} \right)^{-\theta(l-2)} \frac{1 - 2\rho - E}{1 - \rho} \frac{\rho - 1 - E}{\rho + E} \left(\frac{\rho - 1 - E}{2\rho - 1} \right)^{l} \theta(l - 1). \tag{A.4}$$

If one maximizes the entropy with respect to E, then $E = -\frac{(1-2\rho)^2}{\rho}$ and the calculations with only ρ fixed are recovered.

A3. Fixing three quantities (M3)

The entropy of metastable states per site is

$$\begin{aligned} s(\rho, E, \alpha) = {} & (\rho - 1 - E)\log(\rho - 1 - E) + (1-\rho)\log(1-\rho) \\ & - (\rho - 2 - E + \alpha)\log(\rho - 2 - E + \alpha) - 2(1-\alpha)\log(1-\alpha) \\ & - (\rho + E - 1 + \alpha)\log(\rho + E - 1 + \alpha) \end{aligned} \tag{A.5}$$

and the distribution of domain lengths is exponential only for $l \geqslant 3$:

$$\begin{aligned} P(l) = {} & \frac{\rho + E - 1 + \alpha}{(1-\alpha)(1-2\rho-E)}\left[\frac{(\rho-1-E)(\rho+E-1+\alpha)}{(\rho-2-E+\alpha)(1-2\rho-E)}\right]^{\theta(l-2)} \\ & \times\left[\frac{(1-\alpha)^2}{(\rho+E-1+\alpha)(\rho-2-E+\alpha)}\right]^{\theta(l-3)}\left[\frac{\rho-2-E+\alpha}{\rho-1-E}\right]^{l}\theta(l-1). \end{aligned} \tag{A.6}$$

Here again, the value of α which maximizes the entropy is $\alpha = 1 - \frac{(\rho-1-E)(\rho+E)}{2\rho-1}$ and with this value the calculations with ρ and E fixed are recovered.

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